Some Issues in the Development of Computer Art as a Mathematical Art Form

Richard Wright

ABSTRACT

In this paper the author considers some of the issues that arise when mathematics is used to make art (predominantly visual art), in particular the possible conflicts between the role of the mathematician as artist and of the artist as mathematician. Mathematics in art can be approached in a number of ways, as analyses and 'simulations' of artworks and processes perhaps by artificially intelligent systems, as 'ready-made' mathematical objects appropriated by an artist, or as products of the creative imagination in their own right. These approaches are examined and criticised, and connections are made and used to highlight the difference between the mathematics *of* art and mathematics *as* art. The relevance of ideas in the theoretical history of computing and philosophy of mathematics is revealed and used to open up a critical context for this kind of computer art.

Mathematics has been an activity of crucial importance in human thought for many centuries, and no more so than now in this computer age. Yet its ways of thinking often seem an anathema to artistic values, its products remaining aloof, alien and detached from experience. Mathematics has also tended to stand apart from the empirical sciences. It does not seem to involve the same kind of inductive reasoning and the testing of theories against observation, but appears to be self-sufficient. In the great debate between the Rationalist and Empiricist philosophers of the eighteenth century, mathematics was the prime example of the human mind's ability to construct abstract theories of great power from pure deductive reasoning. Even when Kant tried to unite these tendencies of Western philosophy, he preserved the role of mathematics as the keeper of *a priori* logical truths.

"Mathematics is the science that draws necessary conclusions" was the definition attributed to mathematician Benjamin Peirce in 1881; it was a view echoed by many thinkers at the turn of the century [1]. Indeed mathematics does have this quality of certainty about it: its theorems follow inevitably from self-evident assumptions of axioms using the logical laws of non-contradiction. This feeling of completeness due to the conception of mathematics as a closed system causes many people to be surprised that there is such a thing as the creation of new mathematics at all.

The aim of this article is to reach an understanding of the nature of mathematical activity, a definition that might be useful to computer artists. We begin by asking what mathematics and art, as descriptions of the world, have in common. Firstly, how rational is art, is it a logical sort of process that just produces pictures instead of theorems, and can it therefore be appreciated in those terms? The answer to these questions will be neither yes nor no, but the discussion will be used to probe further the realms of mathematics and art.

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Received 27 April 1988.

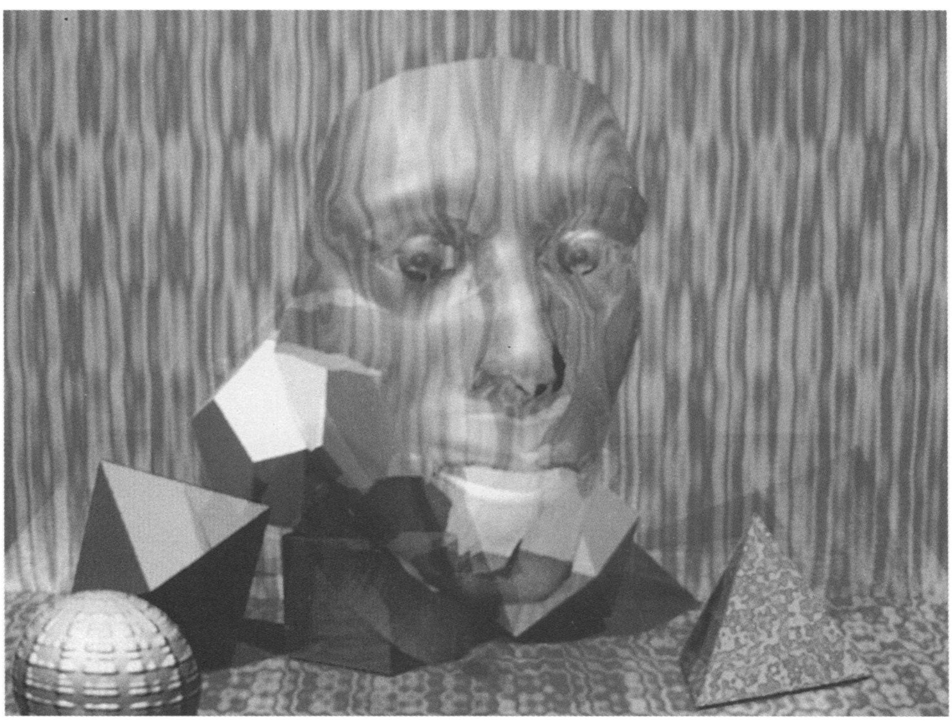

Fig. 1. Richard Wright, *The Disembodied Intelligence*. Face Model by Keith Waters. Software: artist's own software in 'C'. Hardware: VAX 11/785, Gems Framestore, Dunn Film Recorder. Format: 35mm slide of computer-generated image, 1988. The computer as a model of human intelligence is one essentially detached from its environment, existing somewhat out of context. In this image the familiar human visage is surrounded by objects symbolising the results of intelligence abstracting from the world it interacts with. These include the five regular Platonic solids as well as an irregular bump-mapped sphere and textured background. The face is rendered as transparent, to give a sense of both reality and unreality: an ethereal consciousness floating in a private world of mental constructs.

Pergamon Press plc. Printed in Great Britain.
0024-094X/88 $3.00+0.00

Mathematical models view the world as a sequence of precisely related events, and one might question whether art falls into this category or even if it is a relevant approach at all. Because art is regarded as reflecting the deepest experiences of the human mind, the mathematics of art might on the one hand imply a far-reaching explanation of the nature of psychological processes; on the other hand it might contribute just another theoretical tool to art-making, like colour theory did at the end of the nineteenth century.

THE COMPUTER AS ARTIST

As part of the effort in recent years to explain aspects of human activity in terms of mathematical and computational processes, art and algorithmic aesthetics have received a certain amount of attention [2]. This has often taken the form of analysing existing artworks in order to reveal some mathematical structure in them, a structure perhaps not consciously intended by the artist but forming some subliminal deterministic basis to his or her artistic decisions. Before the advent of artificial intelligence research, the idea that aesthetic appreciation might have an underlying mathematical explanation was believed by many writers who sometimes conducted dissections of classical paintings to find some compositional format [3]. Just as the language of musical harmony was derived from geometrical ratios—legend has it by the philosopher/mathematician Pythagoras—geometry has often seemed an obvious starting point for a mathematical analysis of fine art, and developments this century have been no exception. Many of these attempts have since been criticised as being gratuitous, although it is well known that particular artists, especially of the early Renaissance, and some composers have used geometrical proportions as compositional aids [4].

The implication of much artificial intelligence work, however, is by extension to show that the mental processes going on inside the artist's head are also of an algorithmic nature and may also express themselves in artworks in the form of various mathematically defined characteristics. In order to be able to recognise a 'successful' creation of a work of art by a machine without having to wrestle with the philosophical problems of defining art explicitly, these scientists normally use a behavioural definition. It amounts to saying that if an art object produced by some 'artificial' method is generally accepted by its spectators as a genuine work of art, then it is so defined. Some early computer art was produced in a similar fashion by formulating a set of generative rules derived from a particular artist's style and then making them act upon a group of simple pictorial elements (e.g. A. Michael Noll's computer-generated 'Mondrians' [5]). The resultant images are then compared to actual examples of the origi-

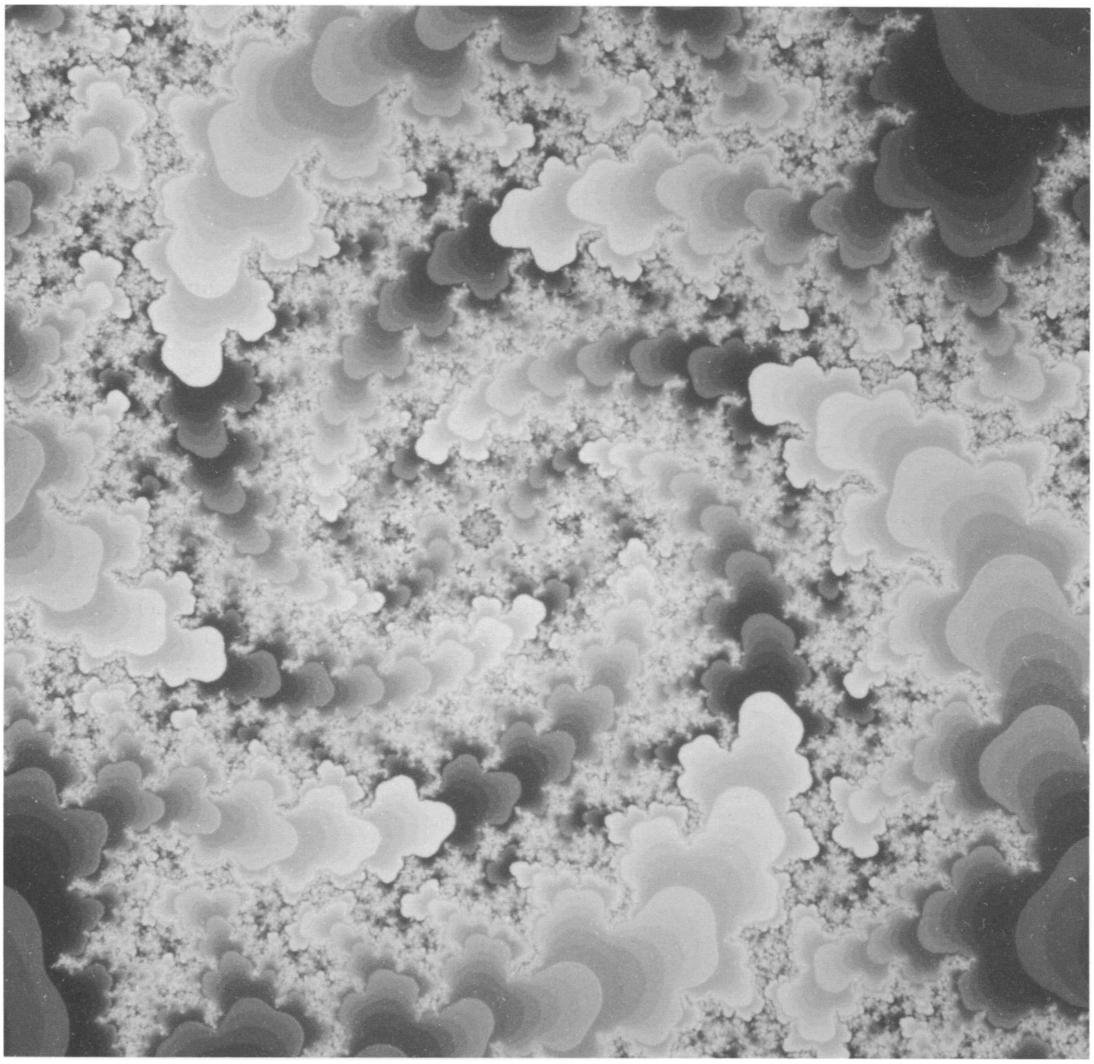

Fig. 2. Richard Wright, *Image from the Mandelbrot Set*. Software: artist's own software in Pascal. Hardware: IBM 4041, IBM 5080 Display. Format: photographic print of computer-generated image, 1985. An example of 'Map Art', mathematical objects produced and studied by mathematicians and also exhibited as art.

nal artist's work. More recently, and with substantial success, Harold Cohen has sought through his artificially intelligent program AARON to simulate the pictorial forms of his earlier painting style [6]. Though the program begins with basically random elements, it is guided by a highly sophisticated system of aesthetic rules that Cohen has built up by carefully observing his own artistic methods and preferences.

Instead of being the results of autonomous art-making, though, these works might be seen as an extended form of reproduction, of a pictorial style. What is the point of these mathematical 'forgeries', of computer-automated art, filling the world with plotter drawings like the model T Ford? The possibility of the computer as an 'original' artist does not in itself tell us much about what to expect from such art, what issues it would address. Although these criticisms might appear premature, the most obvious difference between this behavioural approach to making art and what artists actually do is something in philosophy called intentionality—it concentrates on *how* art is produced rather than *why* [7]. Attempts to explain why an artist might have chosen a particular style or what the artist hopes to achieve by engaging in this occupation are not considered relevant to the computer model. But is not this sort of criticism also vulnerable to the same argument that is used to construct it? Namely, that it in turn ignores the reasons why this attempt to re-create a work of art was undertaken by computer scientist/artists in the first place. Perhaps their implied belief that nature is mathematically determined is as good an artistic reason as any, though this would mean their having to include themselves and their motives in their model of art. So in what sense could a computer model an artist's behaviour? This leads us to a consideration of mathematics as a system of representation and of its limitations, especially as regards the problems of artificial intelligence.

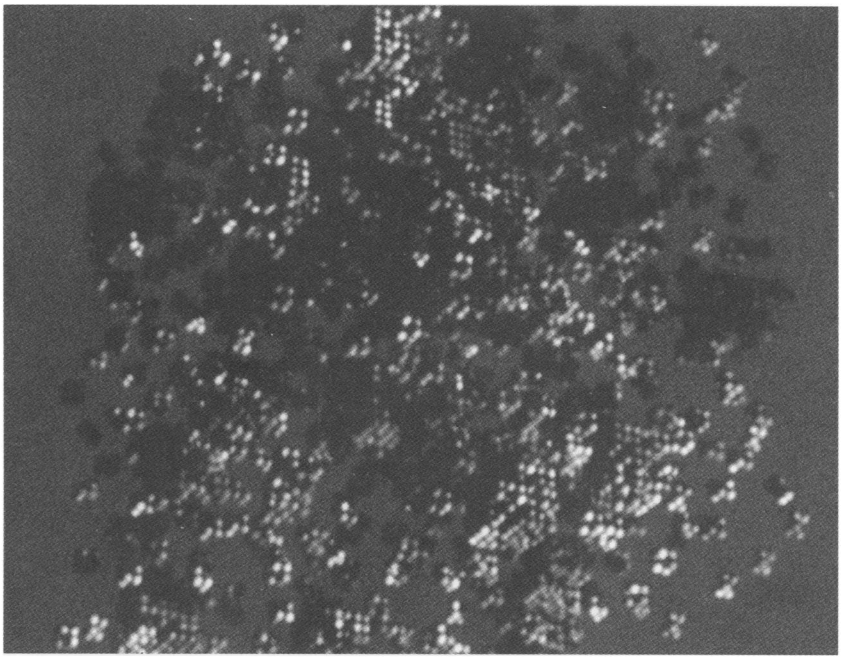

Fig. 3. Richard Wright, *Cellular Object No. 3*. Software: artist's own software in 'C'. Hardware: VAX 11/785, Gems Framestore, Dunn Film Recorder. Format: 35mm slide of computer-generated image, 1987. A set of data has been generated by a cellular growth algorithm, modeled using spheres and rendered using ray-tracing. The object is composed of several thousand individual particles, points picked out of a 3-D lattice, forming clumps of varying shapes and sizes. They were coloured by a solid texturing function to suggest an alternative form of coherence for this imaginary structure.

AI and Problems of Semantics

In his paper that has come to provide so much of the theoretical justification for artificial intelligence research, Alan Turing introduced a definition of intelligence by which machine intelligence could be assessed [8]. Turing proposed an operational definition of thinking called the Imitation Principle. He described a game in which an interrogator would have to decide which of two people, a man and a woman, was the woman on the basis of written replies to questions. Both were allowed any method of persuasion except practical demonstrations to try to convince the interrogator that they were the woman. The point was that the successful imitation of the woman by the man would not prove anything, because gender was based on physical facts not reducible to symbols. In contrast to this, Turing argued that the method *would* apply to intelligence, so that a computer would be displaying intelligent behaviour if the interrogator could not tell a computer apart from a man. There was no way of judging whether people were thinking intelligently other than by comparison with oneself, and he saw no reason to treat computers any differently.

As to whether the human senses, muscular activity and bodily chemistry were relevant to thinking, Turing wrote:

> It will not be possible to apply exactly the same teaching process to the machine as to a normal child. . . . one could not send the creature to school without the other children making excessive fun of it. . . . We should not be too concerned about the lack of eyes, legs, etc. The example of Miss Helen Keller shows that education can take place provided that communication in both directions between teacher and pupil can take place by some means or other.

For his proposed subjects for automation Turing chose only those that involved no contact with the outside world—chess, mathematics, cryptoanalysis, anything that was primarily a matter of technique. This approach assumed that the physical characteristics of the brain and body had no direct bearing on intelligent activity, that it was possible to abstract the essential properties of thinking and limit them to symbol manipulation. But the purpose of intelligent behaviour is to guide the human organism through its dealings with the outside world. Can there be such a thing as a disembodied intelligence, detached from its environment and existing somewhat out of context? Should a discussion of intelligence be limited to what goes on inside our heads or must it include an organism's entire way of life? Intelligence operates in order to effect changes in the world in which it lives; otherwise it is a meaningless game, devoid of a *raison d'être*.

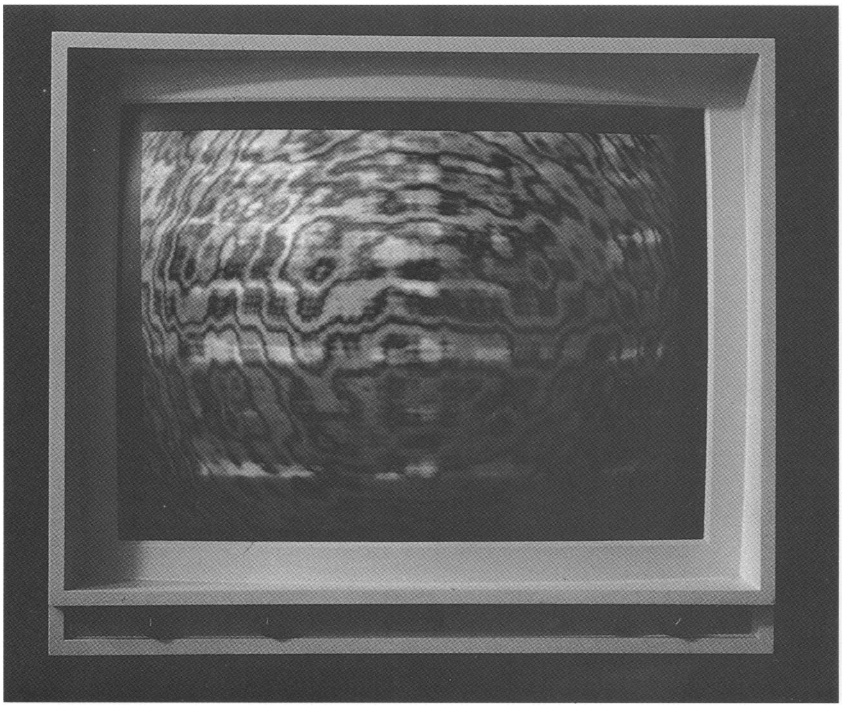

Fig. 4. Richard Wright, *Window*. Software: artist's own software in 'C'. Hardware: VAX 11/785, Ikon Framestore. Format: 35mm slide of computer-generated image, 1988. Two surfaces have been rendered in close-up in order to create an ambiguity between the image as a representation of a solid object and the physical reality of the picture plane. A transparent cover is shielding us from another surface beyond, like a view through the glass of the screen.

Turing anticipated some objections to his thesis:

> May not machines carry out something which might be described as thinking but which is very different from what a man does? This objection is a very strong one, but if we can construct a machine to play the imitation game we should not be troubled by this objection.

In any case the disembodied Turing machine would naturally display an intelligence very different from human intelligence. Building a thinking machine might be as appropriate as trying to breed a flower that barked like a dog.

At first it might seem justified to build a behavioural representation of the artistic process that does not include artistic motivation—as long as the computer system is capable of making objects that other people can respond to as art then one has indeed succeeded in producing bona fide artworks. Supposedly no one would need to know why or even how they had been made. But to concentrate attention on the art object in its final state without regard to the context in which it was made would be to neglect the function of artistic endeavours. Normally it is difficult to see how it can make sense to talk of 'simulating' or 'modelling' art, because the model always tends to become part of the artwork itself. If we see a computer-simulated Mondrian next to a 'real' Mondrian, for instance, then obviously much of the meaningfulness of that comparison is generated by the knowledge that one was created by a human and the other by a machine. To take this one stage further, if the spectator is deliberately prevented from gaining knowledge of how the work was created, or given false information regarding this, then it would seem wise to regard part of the content of the resulting artwork as being determined by the motive behind this act of deception. To doom the spectator to perpetual ignorance concerning the true origins of what is being presented as art is to deny one of the great objectives of art: a means of gaining self-knowledge. How much of ourselves would we see reflected in an 'autonomous' computer's art? Would we feel sympathy with it or alienation?

A mind is a bit like a stone falling into a pond and sending ripples travelling out all over the surface and bouncing back again from the banks. We might point to the center of the ripples where the stone first struck and say 'Here is a mind', but we must also look beyond this to the undulating surface where the interference of many drops is apparent. A computer is then like a plastic beaker half filled with water; it can gently bob up and down to the rhythm of the waves and its contents are also the constituents of the pond water, but it is essentially sealed from its environment. To accurately model the complex dynamics of turbulent flow, research suggests we need an explicit simulation rather than to study particles in isolation [9]; perhaps to build a really human-like mind it would be necessary to model a whole society of minds. But the computer-mind, of course, is already functioning in a society of minds—the human society that spawned it—and it can be semantically grasped only with respect to that social context.

It has been suggested that we can either use the computer in a premeditated way for a particular end like any other medium or tool, or else leave it to operate without human interference as much as possible, presenting the results later, for the viewer to provide semantic content [10]. But we cannot unload our artistic responsibilities onto the computer quite so easily as this. If we agree that art is a language and that its functioning depends on our sharing a common cultural context, then we must concede that an artist and his or her public do not operate in isolation from one another. To function as an artist, an artificially intelligent machine would have to be aware of the world of social intercourse, but this does not mean that it cannot produce art, as long as we recognise the wider significance of the computer in human affairs as part of the context in which the artwork is understood.

The use of computer-generated art as a way of revealing and exploring some intrinsic language of the machine itself is to suggest a rather more independent and objective relationship between human and machine than seems justified. But this idea that mathematical systems have a 'life of their own' is something that we shall return to later. We now turn our attention from the computer as artist to the mathematician as artist, to see if an examination of mathematical research can tell us a little more about the mathematical models that are implemented on and that define the operation and nature of the computer.

Platonism and Formalism in Mathematics and Art

Most mathematicians feel themselves to be discovering true and objective facts about mathematical objects (i.e. Platonism [11]), while artists have always been seen as the individual creators of what they produce. If we accept this view of mathematical activity, then artists working with mathematics (presenting mathematical objects as art) are put in the position of appropriating a kind of mathematical 'ready-made' by placing it in an artistic context [12]. But can artists legitimately feel they are able to create their 'own' mathematical objects? Are they not rather in the position of selecting certain structures that already exist conceptually in an external mathematical world?

Probably the most successful recent exhibition of mathematical art was 'Map Art', images of iterative mappings in the complex plane, produced by a team of mathematicians and physicists at the University of Bremen in West Germany [13]. These images were colour plots of the parameter spaces and dynamics of some non-linear functions, particularly of the simplest one called the Mandlebrot set. They were created not with artistic issues in mind but for mathematical research and for their interest as mathematical objects. (It is still a common presumption that geometric complexity is equivalent to semantic complexity; this tends to promote the view that pictures of dynamic systems are more 'artistic' because of their visual irregularities. This idea, however, is misleading. Islamic art, for instance, is an expression of a vast religious and cultural belief system, but when taken out of context it appears to many Western eyes as wallpaper patterns.) Visually the images are striking; they seem to exhibit a fairly ordered nesting of catherine wheel spirals and paisley patterns but with an infinite degree of detail that one would not normally associate with regular forms. They do not appear 'mechanical', but are still too precise to be manufactured by human hands. When we come to confront them as art, how are we to come to terms with their existence as mathematical phenomena with objective properties, seemingly independent of their discoverers, almost like the products of the artificially intelligent art machines discussed above?

Variations of this impersonal aspect of mathematical art emerge when we consider the question of authenticity. When a mathematician or scientist has published the results of his research then that work seems to become the common property of the scientific community. It is not appropriate to try to pursue ownership rights over a law of nature. Mandelbrot did not copyright his set of points so that artists would not be able to use them. It might be argued that although no individual can claim exclusive right to a scientific theory that is part of the general intellectual achievements of humankind, he or she could take out a patent on a particular *application* of that knowledge. Likewise an artist might share his ideas about art with his fellows and examine the work of others, but any particular painting or sculpture that he executes is his and his alone. But the situation is not always this straightforward. Consider an artist (or mathematician) using a mathematical function to generate images that he intends to exhibit as art. While he is at lunch another 'artist' comes into his studio/laboratory and generates a completely different image just by tweaking one of the parameters of the function. Is this small act enough to warrant the intruder as the creator of a new work? And in any case does the original artist have any more right to a mathematical object whose existence might seem as objective as the sun in the sky? In addition, due to the mechanical or electronic means of production, no appeal to individuality can be made by emphasising any stylistic attributes caused by a traditional manual execution of the picture. If an artist engages in mathematical research, can this in any sense be an artistic activity? If the artistic act lies mainly in the appropriation of a particular mathematical object into an artistic context, we are close to saying that mathematics cannot in itself be used as a language for art and that the artist's role is little more than that of a selector.

It is difficult to explain the power of mathematics when applied to the external world except by an appeal to some kind of objective existence. For example, before the First World War David Hilbert had developed a generalaistion of Euclidean geometry which involved a space of infinitely many dimensions. Later John Von Neumann used these 'Hilbert spaces' to make precise the idea of the state of a quantum-mechanical system like an electron in a hydrogen atom. Likewise in 1932 physicists discovered the positron, whose existence had been predicted some years before by P.M. Dirac on the basis of an abstract mathematical theory. The expansion of pure mathematics for its own sake has often borne unexpected fruit in science [14].

No one today would try to demand the empirical justification of different mathematical systems such as the 'truth' of Euclidean or non-Euclidean geometry, and mathematicians are free to choose between the two. Since the nineteenth century, mathematics has come to be seen more and more as a creation of the human mind, and Platonism as a philosophy has declined. Platonism had originally been a belief in the intuitive truths of the axioms of Euclidean geometry, but after the discovery of non-Euclidean geometry and the existence of counter-intuitive objects such as in Cantor's theory of infinite sets, there arose a concern that intuition could not be trusted [15]. Mathematical objects were to be considered valid only if they could be derived rigourously by logical deduction from a set of axioms. Perhaps the logical basis of mathematics guaranteed its correspondence with the orderly laws of nature.

At first there was an attempt to reduce the foundations of mathematics to set-theoretical logic (e.g. Frege, Russell et al.), but when that proved too problematic mathematicians turned to place their faith in the logical consistency of language itself. The resulting philosophy formulated by David Hilbert was called Formalism, and although his ultimate goal of proving the consistency and completeness of mathematics as a formal system was shown by Kurt Gödel not to be possible, it became the dominant foundationist dogma. Formalism avoided the Platonic absolute character of mathematical existence and gave mathematicians the freedom to explore alternative axiomatic systems, but by concentrating on the logical syntax of the language it denied that mathematics was 'about' anything and tended to empty mathematics of meaning [16]. This century many Constructivist artists have pursued 'formal relationships' [17], but this has led to the appearance of a certain sterility in their work and an aversion to mathematics by many other artists.

The formal analysis of mathematical language led to the idea that if mathematical propositions were derived logically and consistently from the axioms in a mechanical fashion, then maybe the process could be completely automated. This resulted in the 1930s in the beginnings of computer science in the form of the Turing machine, the only machine conceivably powerful or general enough possibly to be able to solve any mathematical problem put to it. But rather than reach any final conclusion on the limits of mathematics, the project launched a whole new field of study in mathematical logic and the theory of automata, emphasising the open-ended nature of the issues it was developed to settle once and for all.

It can be a source of amusement to ponder the total number of images that, theoretically, could be produced on a digital framestore. As a problem in combinatorics it is easily solved by raising the total number of colours available in one pixel by the total number of pixels addressable. Even for a framestore of moderate capacity this number is immense (16 million raised to the half million would be fairly common), and this number is usually very much larger than the total number of particles in the known universe (about 10 to the 80). It is interesting to speculate on the advantages of working through and classifying this set of all possible images—it would include the faces of everyone who ever lived, a page or two of text from every book ever written, a copy of every painting executed and all possible variations of each, as well as all sorts of mathematical graphics and diagrams. But supposing the number of all possible pictures in digital form was much smaller than this, only about a thousand, say, and supposing someone generated all these pictures and exhibited them in a large gallery. What would this mean? Would it mean that they had solved all problems of the plastic arts? No, because it makes no sense to talk of solutions without a clear understanding of the problems. It is rather like an ultra-Formalist philosopher of mathematics who believes that all mathematical theorems are just combinations and permutations of symbols, or that painting is merely the business of placing marks on a canvas.

Turing was a Formalist and tended to treat mathematics as a game without connection to the outside world. He avoided explicitly defining what intelligence really was, just as Formalism avoided asking what mathematics really was by reducing it to a system of formal rules. In the Turing model, thinking became the activity of shuffling abstract symbols; this might be described as not so much the ability to think as the ability to dream [18].

By the mid-twentieth century Formalism had become the official philosophy of mathematics, although most practicing mathematicians were still Platonists in that they believed they were discovering true facts about real mathematical objects. Formalism was linked to Logical Positivism, the philosophy of science that rose to dominance in the 1940s and 1950s, and which has lingered on in the absence of anything definite to replace it. Its goal was a unified science expressed in a formal logical language and with a single deductive method. In order to relate theory to experiment, rules were devised for the interpretation of results, rules of physical measurement, mass, length and time. Mathematics is viewed as the language in which scientific theory is formulated and enveloped, with no independent subject matter of its own.

The heritage of Frege, Russell and early Wittgenstein left a school of Analytical Philosophy in which the central problem is the analysis of meaning using the logical syntax of language. The philosophy of mathematics was identified with the study of logic and formal systems, making an account of its historical and pre-formal development an irrelevance [19].

In 1934 there was a revolution in the philosophy of science when Karl Popper proposed that scientific theories are not derived simply by inductive logic from experimental observations, but are invented hypotheses which are then subjected to critical analysis. A theory is scientific if it is capable of being tested and refuted—if it survives it attains some degree of credibility but can never be proved. In the 1960s Imre Lakatos decided to apply this approach to the philosophy of mathematics.

MATHEMATICS AS CULTURE

Lakatos' major work, *Proofs and Refutations,* describes a classroom dialogue [20]. The students are made to take the parts of various historical figures in mathematics as they try to find a final version of Euler's law $V - E + F = 2$ for solid polyhedra. During the resulting discussion, different pupils put forward different theories that they attempt to prove and disprove by argument and counter-examples. At the same time Lakatos details the corresponding historical events in a series of footnotes, showing the role that proof plays in the development of mathematics and the formation of concepts. Lakatos uses this representation of history to show that mathematical knowledge grows like natural science, by the continual criticism and correction of its theories. 'Proof' in this context does not mean the mechanical process by which theorems are derived from axioms; it is a method of explaining new ideas more fully, of justifying and elaborating them. Lakatos describes the use of proof and logical deduction in mathematical research as it is practiced every day by mathematicians. He uses this to show that the Formalist view of mathematics is an abstraction that is hardly to be found anywhere outside textbooks on symbolic logic. Lakatos argues that the dogmatic foundationist philosophies of mathematics are untenable because of their inability to accept the informal growth of mathematics as the basis of mathematical knowledge.

What gives mathematics its descriptive power is its close relationship to other areas of human thought—because it is consistent with our culturally defined beliefs [21]. Similarly, empirical science is seen to be successful simply because we are in a society which places high value on the areas in which the scientific method is appropriate. The search for solid foundations for mathematics in logic was like the need of a subculture to strengthen its own identity. After it was discovered that logic was not a unique theory, it became another branch of mathematics. Mathematical concepts are like cultural artifacts, continually expanding in a way that prevents any final definition or perfect rigour. The meaning of mathematical objects lies in the shared understanding of human beings, not in external reality. In this respect it is similar to an ideology, religion or art form; it is intelligible only in the context of human culture. Having said this, can we get an idea of what cultural or artistic values could be identified in a mathematical art?

MATHEMATICS AS COMPUTER ART

The only objects that can be studied visually are ones that can be 'constructed', that is, classes of mathematical objects of which an actual example can be constructed (in contrast to objects whose only reason for being is that it would be logically contradictory for them not to exist). Here then is a movement away from dialectical and existential mathematics, towards the concrete and algorithmic. Indeed, Lakatos describes his philosophy as 'quasi-empiricist' [22].

In mathematical research, graphics generally are used to explore the structure of an object and to reveal properties not so immediately apparent using other means. Any visually perceived regularities can then be followed up by more rigourous methods of study. One example of this is in the field of research into cellular automata [23]. To some mathematicians it seems inappropriate to make mathematical judgments based on graphical representations ('Pictures don't prove anything'). But it is strange to think that the very first proofs were prompted by geometric forms—Thales' theorem that a circle is bisected by its diameter is the first recorded, in about 600 BC, and it is difficult to see how he would have been inspired to form this concept without the stimulus of his diagrams. This intuitive certainty of the properties of the visual world seemed to justify the construction of the axiomatic system and the deductive method itself. It was appreciated by the ancient Greeks that some facts of number theory could be more easily proved by representation as geometrical figures [24]. Since then the reverse has tended to be the norm. Geometrical intuition came to be distrusted in the nineteenth century, partly due to the discovery of fractal 'monster' curves whose analytical properties seemed to defy geometrical sense [25]. These contentions were resolved by the re-definition of geometry using point sets in the 1930s, but by then the search for foundations in mathematics had shifted focus. It would be unfair, however, to blame this episode on any supposed inadequacies of perception rather than on the inability of analysis to model the human visual system.

Proof is only one tool whereby mathematicians progress in their understanding of the objects they study, and it is meaningless without regard to the current state of mathematical consciousness. There is no reason to devalue properties as being irrelevant to the aims of mathematics. This is to say that both mathematicians and artists have cause to be interested in graphical representations of the conceptual forms of mathematics. Graphics are used as a method of visual thinking similar to a designer sketching out his or her ideas; they are not just to communicate information.

There is now a tendency to view mathematical formulae as processes to simulate phenomena rather than to describe their structure explicitly. It is becoming clear that general laws to

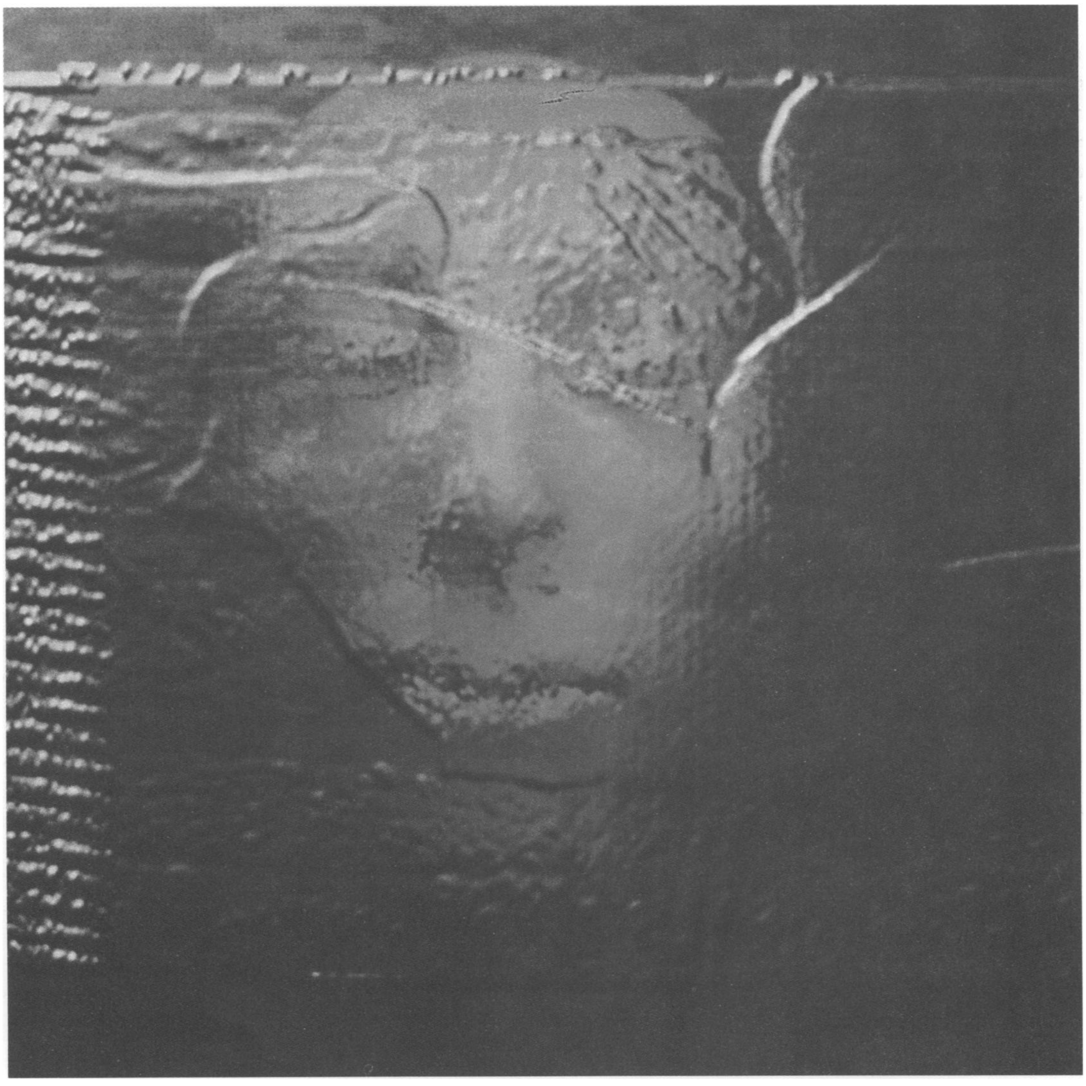

Fig. 5. Richard Wright, *Medusa.* Face Model by Keith Waters. Software: artist's own software in 'C'. Hardware: VAX 11/785, Ikon Framestore. Format: 35mm slide of computer-generated image, 1988. A nude from a pin-up magazine has been digitised and converted into a bump-map. It forms a stony relief over the plane of the screen in which there is seen the dispassionate reflection of a mask. This image might be the face of a spectator being confronted with the surface as a barrier, or an attempt by the computer to represent the environment external to the picture and build a more direct relationship with the world of humans.

describe the behaviour of many natural systems may not exist, and that they can only be studied by direct simulation [26]. Once the system has been constructed it can be studied as an object in its own right. This stresses the creative aspects of mathematics and the importance of using all humanity's perceptive faculties; it is a move away from analysis to synthesis. Systems with an unpredictable character might find more personal resonance in a spectator. But even with unpredictable dynamics, it would still take an act of creative perception to recognise their characteristics as significant or meaningful, rather than just autonomous facts. Fractal curves were around for over a century before their relevance was discovered (or created). It has been said that mathematics can generate 'unimaginable' forms, but these forms must be imbued with some personal relevance by their instigators in order to merit any attention at all.

There is no mathematics of art, only mathematics as art. Any mathematical model becomes the content of the artwork. It is used to create, and mathematics can form the subject of art by virtue of its function as an expression of human sensibilities. Mathematicians are free to build theories and to make pronouncements about them, but this activity is not arbitrary. Their development must satisfy some cognitive need, and it is in the wider context of such needs that the artist operates. Our humanistic definition of mathematics allows a more artistic evaluation of mathematical concerns—the quality of the existence of mathematical objects like numbers, the nature of mathematical truth.

Issues in the philosophy of mathematics can be generalised to the artistic arena. It is appropriate to subject the products of mathematical research to all the methods that are usually applied in order to come to terms with cultural artifacts like art, the tension between objectivity and subjectivity, their metaphorical meanings and the character of representational systems.

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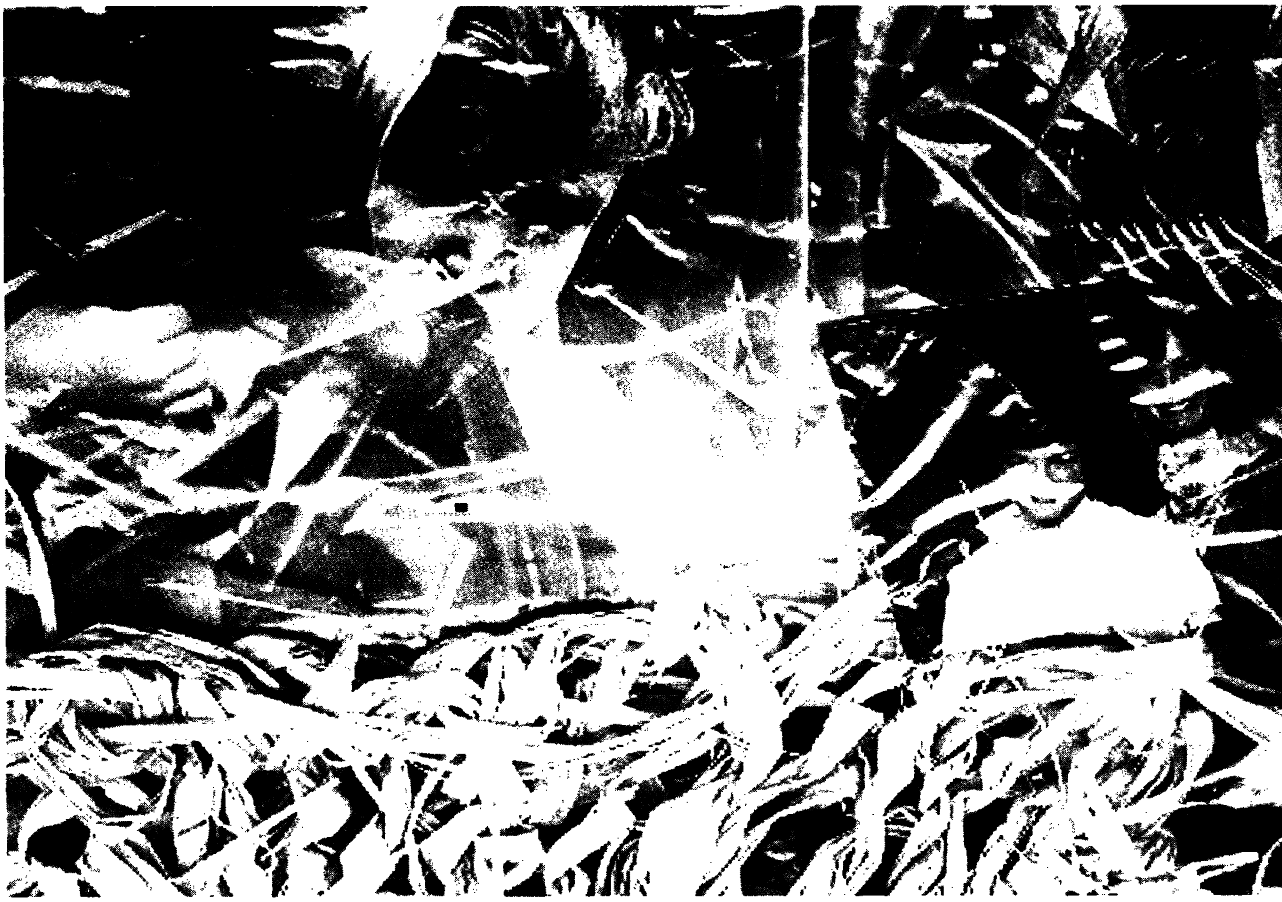
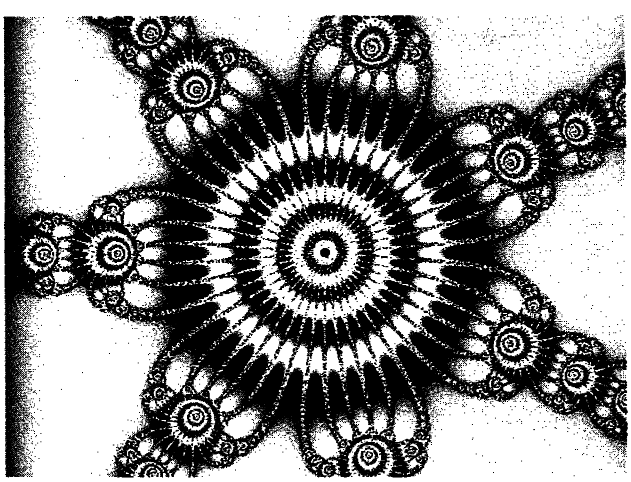
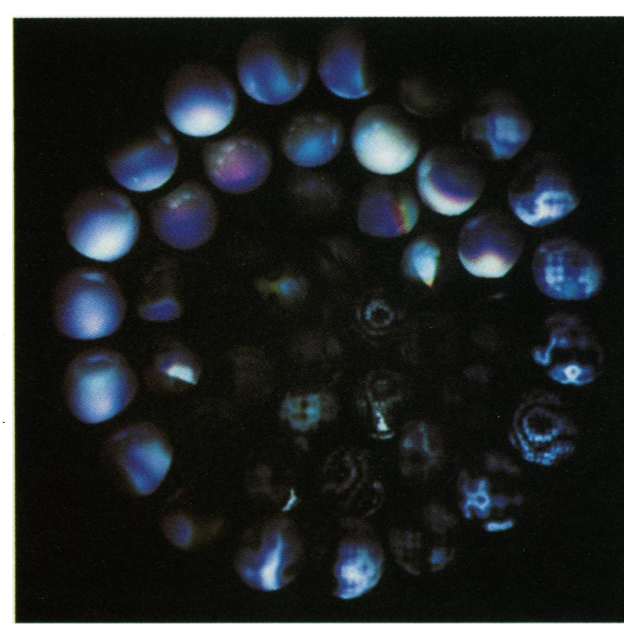
Editor's Note: The reader is referred to Color Plate A No. 3 for an illustration by Richard Wright.







Color Plate A

No. 1. Top. Joan Truckenbrod, *Time Knit*, digital photograph, 24 × 26 in, 1988.

No. 2. Bottom left. Brian Evans, fractal image created using Newton's method for finding roots of the equation $f(z) = z^7 - 1$. The RGB triplet measure for this image is 1:1:1 with total intensity at half of full.

No. 3. Bottom right. Richard Wright, *Parameter Space*, software: artist's software in 'C'; hardware: VAX 11/785, Gems Framestore, Dunn Film Recorder; format: 35-mm slide of computer-generated image. 1987. A fractal sine function was used to solid texture map a conical arrangement of spheres. Computer algorithms can take arbitrary sets of data and fuse them together to create an object that possesses the quality of tangible reality.